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Supertwistors as quarks of SU$(2,2|4)$

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Abstract: The GS superstring on $AdS_5 \times S^5$ has a nonlinearly realized, spontaneously broken SU$(2,2|4)$ symmetry. Here we introduce a two-dimensional model in which the unbroken SU$(2,2|4)$ symmetry is linearly realized. The basic variables are supertwistors, which transform in the fundamental representation of this supergroup. The quantization of this supertwistor model leads to the complete oscillator construction of the unitary irreducible representations of the centrally extended SU$(2,2|4)$. They include the states of $d=4$ SYM theory, massless and KK states of AdS_5 supergravity, and the descendants on AdS_5 of the standard massive string states, which form intermediate and long massive supermultiplets. We present examples of long massive supermultiplets and discuss possible states of solitonic and (p,q) strings.

Keywords: Superstring Vacua, Space-Time Symmetries, Conformal and W Symmetry, Global Symmetries.

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Contents

1. Introduction

Supertwistors have not yet been fully incorporated into the study of the AdS/CFT correspondence [1]. Just as Penrose twistors [2] carry the fundamental representation of the conformal group, Ferber's supertwistors [3] carry the fundamental representation of superconformal symmetry. In this paper we shall restrict ourselves to $\mathcal{N} = 4$ superconformal symmetry in $d = 4$, which has the superalgebra $\mathrm{SU}(2, 2|4)$. This is also the $\mathcal{N} = 8$ AdS algebra in $d = 5$. Supertwistor variables, upon which $\mathrm{SU}(2, 2|4)$ is realized linearly, may also give a natural framework in which to study extended objects on spaces with this symmetry.[1]

In this note, we will present a 1+1-dimensional supertwistor model. The supertwistor variables will take the role of 'quarks' for $\mathrm{SU}(2, 2|4)$, generating copies of the fundamental representation in the Fock space of states. Using these, we derive dynamically the complete set of positive (conformal) energy unitary irreducible representations (UIR's) of $\mathrm{SU}(2, 2|4)$ given in [5, 6] from the general theory of oscillator construction of the representations of noncompact groups [7] and supergroups [8].

[1] For instance, [4] recently studied the superparticle in conformal superspace with additional tensor charges using these variables. This paper also contains a list of important references to twistors and supertwistors in addition to those used in the present context.

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This set includes:

Doubleton supermultiplets that have spin range between 2 and 4. The CPT–self-conjugate doubleton supermultiplet is the $\mathcal{N} = 4$ Yang-Mills multiplet in $d = 4$ and corresponds to gauge degrees of freedom that couple only on the boundary of AdS_5.

KK supergravity multiplets including both the $\mathcal{N} = 8$ graviton supermultiplet on AdS_5 and the massive Kaluza-Klein modes coming from S^5 compactification of type IIB supergravity. They all have spin range 2.

General massless supermultiplets[2] on AdS_5 with spin range between 2 and 4, and

'Novel' short supermultiplets which have a non-vanishing value of the central charge Z in the centrally extended $\mathrm{SU}(2,2|4)$ and are conjectured to be part of the (p,q)-string spectrum. They have spin range 2.

In addition to the above supermultiplets with maximum spin range 4, already described in [5, 6], we will focus here on the massive supermultiplets of $\mathrm{SU}(2,2|4)$ which originate from

Massive string states with maximum spin range 8.

The existence of this construction follows from the fact that the model has two independent supertwistors (corresponding to left-moving and right-moving excitations) and that independent oscillators exist at each point along the string. The latter condition in particular allows a Fock space state to be built out of arbitrarily many copies of the fundamental ('generations' in the language of [8, 5, 6]; we will use this notation as well). Without this (as in the case of the particle) the spectrum would consist only of the gauge degrees of freedom.

The supertwistor action which we will present here is motivated by that of the GS superstring on $AdS_5 \times S^5$ [9, 10]. This theory has a non-linearly realized, spontaneously broken $\mathrm{SU}(2,2|4)$ symmetry which is inherited from the superisometries of the background space. These superisometries realize the algebra $\mathrm{SU}(2,2|4)$ non-linearly on ten bosonic coordinates (four x along the boundary of the AdS space and six y perpendicular to it, or equivalently five AdS directions and five spherical angles) and sixteen fermionic coordinates θ.

The action of our model, on the other hand, will have a linearly realized $\mathrm{SU}(2,2|4)$ symmetry which is not spontaneously broken. The bosonic part of this action was extracted from that of the $AdS_5 \times S^5$ superstring by restricting it to excitations only of x (in effect taking $\rho \to \infty$ and ignoring the spherical angles). The details of this procedure will be given in a separate paper [11], in which various ways to realize $\mathrm{SU}(2,2|4)$ symmetry as isometries of supercoset spaces will be studied.

In this paper we take the point of view that a two-dimensional action with the simplest linear realization of the SU$(2,2|4)$ symmetry can be interesting without necessarily being derived from any other theory. We will focus here on the fact that one can easily quantize this action, as there are no gauge symmetries but only a global SU$(2,2|4)$.

2. Overview of the super-oscillator construction for the unitary irreducible representations of SU$(2,2|4)$

The general superoscillator construction of non-compact supergroups was presented in [8]. The spectrum of Type IIB supergravity over the 5-sphere was first calculated in [5] and fitted into an infinite tower of short supermultiplets of SU$(2,2|4)$. More recently, all doubleton and massless supermultiplets of SU$(2,2|4)$ were given in [6].

The physically interesting supermultiplets are the UIR's which admit a lowest weight state. For these representations the spectrum of the conformal Hamiltonian (the AdS energy) is bounded from below.

The centrally extended version of the SU$(2,2|4)$ superalgebra (which was denoted U$(2,2|4)$ in [5]) has even subgroup SU$(2,2) \times$ SU$(4) \times U_Z(1)$. The representation space is constructed using the three graded decomposition of SU$(2,2|4)$ with respect to its compact subsuperalgebra SU$(2|2)_L \times$ SU$(2|2)_R \times$ U(1)

$$\begin{aligned}[][L^0, L^\pm] &\subseteq L^\pm\,, \\ [L^+, L^-] &\subseteq L^0\,, \\ [L^+, L^+] &= 0 = [L^-, L^-]. \end{aligned} \tag{2.1}$$

The generators of SU$(2,2|4)$ are given in terms of two pairs of superoscillators

$$\begin{aligned} L^- &= \vec{\xi}_A \cdot \vec{\eta}_M\,, \\ L^0 &= \vec{\xi}^A \cdot \vec{\xi}_B \oplus \vec{\eta}^M \cdot \vec{\eta}_N\,, \\ L^+ &= \vec{\xi}^A \cdot \vec{\eta}^M, \end{aligned} \tag{2.2}$$

where

$$\xi_A = \begin{pmatrix} a_i \\ \alpha_\gamma \end{pmatrix}, \qquad \xi^A = \begin{pmatrix} a^i \\ \alpha^\gamma \end{pmatrix} \tag{2.3}$$

and

$$\eta_M = \begin{pmatrix} b_r \\ \beta_x \end{pmatrix}, \qquad \eta^M = \begin{pmatrix} b^r \\ \beta^x \end{pmatrix}, \tag{2.4}$$

with $i,j = 1,2$; $\gamma,\delta = 1,2$; $r,s = 1,2$; $x,y = 1,2$ and

$$\begin{aligned}[][a_i, a^j] &= \delta_i^j\,, \qquad & \{\alpha_\gamma, \alpha^\delta\} &= \delta_\gamma^\delta\,, \\ [b_r, b^s] &= \delta_r^s, \qquad & \{\beta_x, \beta^y\} &= \delta_x^y\,. \end{aligned} \tag{2.5}$$

Annihilation and creation operators are labeled by lower and upper indices, respectively. The arrows over ξ and η indicate that one takes an arbitrary number P of 'generations' (copies of the oscillators) so that e.g. $\vec{\xi}_A \cdot \vec{\eta}_M \equiv \sum_{K=1}^{P} \xi_A(K)\eta_M(K)$.

To construct a basis for a lowest weight UIR of SU(2, 2|4), one starts from a set of states $|\Omega\rangle$, referred to as the "lowest weight vector" (lwv) of the corresponding UIR. These states are defined in the Fock space of oscillators and transform irreducibly under the maximal compact subsupergroup $\mathrm{SU}(2|2)_L \times \mathrm{SU}(2|2)_R \times \mathrm{U}(1)$. They are defined to be annihilated by the L^- operators

$$L^-|\Omega\rangle = 0\,. \tag{2.6}$$

By acting on $|\Omega\rangle$ repeatedly with L^+, one generates an infinite set of states that form a UIR of SU(2, 2|4)

$$|\Omega\rangle\,, \qquad L^+|\Omega\rangle\,, \qquad L^+L^+|\Omega\rangle\,, \ldots \tag{2.7}$$

The central charge-like $\mathrm{U}(1)_Z$ generator Z which commutes with all generators of the superalgebra PSU(2, 2|4), whose even subgroup is just $\mathrm{SU}(2,2) \times \mathrm{SU}(4)$, is given by

$$Z = \frac{1}{2}\{N_a + N_\alpha - N_b - N_\beta\}\,. \tag{2.8}$$

Here $N_a \equiv \vec{a}^i \cdot \vec{a}_i$, $N_b \equiv \vec{b}^r \cdot \vec{b}_r$ are the bosonic number operators, $N_\alpha = \vec{\alpha}^\delta \cdot \vec{\alpha}_\delta$ and $N_\beta = \vec{\beta}^x \cdot \vec{\beta}_x$ are the fermionic number operators. The basis of SU(2, 2) given above corresponds to the compact basis (with respect to the maximal compact subgroup $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)$). As was shown explicitly in the second reference of [6], one can go to the noncompact $\mathrm{SL}(2,\mathbb{C}) \times D$ (Lorentz group times dilatations) basis and write the generators of SU(2, 2) as bilinears of four component bosonic spinors transforming covariantly under the conformal group. It was pointed out by these authors that the oscillator realization can be reinterpreted in twistor language (section 2).

3. Twistors as quarks of the conformal group

SU(2, 2) is the covering group of the conformal group SO(4, 2). A twistor [2] is a set of two commuting, two-component spinors which linearly realize the conformal symmetries in 3 + 1 dimensions

$$\mathcal{Z}^\Omega = \begin{pmatrix} \lambda_\alpha \\ \mu^{\dot\alpha} \end{pmatrix}; \qquad \alpha,\, \dot\alpha = 1, 2\,. \tag{3.1}$$

Under SU(2, 2), $\mathcal{Z}$ transforms in the fundamental as

$$\mathcal{Z} \to \begin{pmatrix} L - \frac{i}{2}D & K \\ P & \bar{L} + \frac{i}{2}D \end{pmatrix} \mathcal{Z}\,, \tag{3.2}$$

where L and $\bar{L}$ denote the two $\mathrm{SL}(2,\mathbb{C})$ Lorentz transformations, D is the dilatation, K is the special conformal transformation, and P is the translation. These transformations preserve the metric

$$H = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} \tag{3.3}$$

and so we can introduce a conjugate representation

$$\bar{\mathcal{Z}} = \mathcal{Z}^\dagger H = (\bar{\mu}^\alpha, \bar{\lambda}_{\dot{\alpha}})\,, \tag{3.4}$$

such that the bilinear form

$$\bar{\mathcal{Z}}(1)\mathcal{Z}(2) = \bar{\mu}^\alpha(1)\lambda_\alpha(2) + \bar{\lambda}_{\dot{\alpha}}(1)\mu^{\dot{\alpha}}(2) \tag{3.5}$$

is manifestly $\mathrm{SU}(2,2)$–invariant.

The basic idea of [2] is that twistors are fundamental variables which define the space-time coordinates $x^{\dot{\alpha}\alpha}$ via the relation[3]

$$\mu^{\dot{\alpha}} = -ix^{\dot{\alpha}\alpha}\lambda_\alpha\,. \tag{3.6}$$

One easily verifies [3] that the standard conformal transformations of $x^{\dot{\alpha}\alpha}$ follow from (3.2). This suggests that twistors and supertwistors might be useful variables in the treatment of conformal theories.

It is well known that the action for a massless particle in 3+1 dimensions can be rewritten entirely in terms of twistor variables. Since $P_\tau^2 = 0$, we can write $(P_\tau)_{\alpha\dot{\alpha}} = \lambda_\alpha\bar{\lambda}_{\dot{\alpha}}$, and then using (3.6) one finds

$$S = i\int d\tau\, \bar{\mathcal{Z}}\partial_\tau\mathcal{Z}\,, \tag{3.7}$$

or in components

$$S = i\int d\tau\left(\bar{\lambda}_{\dot{\alpha}}\partial_\tau\mu^{\dot{\alpha}} + \bar{\mu}^\alpha\partial_\tau\lambda_\alpha\right)\,. \tag{3.8}$$

This action is manifestly $\mathrm{SU}(2,2)$-invariant. Since (3.7) is quadratic it can be quantized in the usual way by imposing $[q,p]=i$:

$$\left[\mathcal{Z}^\Omega, \bar{\mathcal{Z}}^{\Omega'}\right] = \delta^{\Omega\Omega'}\,. \tag{3.9}$$

In the language of [5, 6] this gives one generation $(P=1)$ of oscillators which can be used to construct the doubleton representations of $\mathrm{SU}(2,2)$.

We would like to relate this to string theory. For this we consider the bosonic sector of string theory on $AdS_5\times S^5$ [10], and truncate out the degrees of freedom associated with the transverse coordinates in the limit $\rho\to\infty$, (i.e. $\partial y = 0$). In the

[3] As usual, we replace four-vector indices with paired spinor indices by $x^{\dot{\alpha}\alpha} = x^\mu\bar{\sigma}_\mu^{\dot{\alpha}\alpha}$.

conformal gauge with light-cone world-sheet coordinates $\sigma_\pm = \tau \pm \sigma$, the action in the first-order formalism becomes

$$S = \int d^2\sigma \left(-\frac{1}{\rho^2}(P_+)_{\alpha\dot\alpha}(P_-)^{\dot\alpha\alpha} + (P_+)_{\alpha\dot\alpha}\partial_- x^{\dot\alpha\alpha} + (P_-)_{\alpha\dot\alpha}\partial_+ x^{\dot\alpha\alpha} + \cdots \right), \tag{3.10}$$

with

$$P_\pm^{\dot\alpha\alpha} = \rho^2 \partial_\pm x^{\dot\alpha\alpha}\,. \tag{3.11}$$

In the boundary limit the constraint equations become $(P_+)^{\dot\alpha\alpha}(P_+)_{\alpha\dot\alpha} = (P_-)^{\dot\alpha\alpha}(P_-)_{\alpha\dot\alpha} = 0$ and can be resolved by setting

$$\begin{aligned}(P_+)_{\alpha\dot\alpha} &= \lambda_{L\alpha}\bar\lambda_{L\dot\alpha}\\ (P_-)_{\alpha\dot\alpha} &= \lambda_{R\alpha}\bar\lambda_{R\dot\alpha}\,.\end{aligned} \tag{3.12}$$

Using eq. (3.6) for the left-movers and for the right-movers we can rewrite the action as

$$S = \int d^2\sigma \left(i\left(\bar{\mathcal{Z}}_L\partial_-\mathcal{Z}_L + \bar{\mathcal{Z}}_R\partial_+\mathcal{Z}_R\right) - \frac{1}{\rho^2}P_+P_- + \cdots \right), \tag{3.13}$$

where $\mathcal{Z}_{L/(R)} = (\lambda_{+/(-)}, \mu_{+/(-)})$ as in (3.2), and $\mu_{+/(-)}$ is related to $\lambda_{+/(-)}$ as in (3.6).

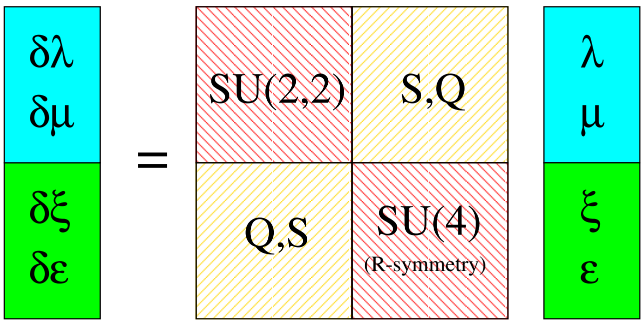
Now we see that in the limit to the boundary we have only the kinetic term for twistors. This free action is *manifestly* $\mathrm{SU}(2,2)$ invariant. Therefore, the theory, being a subset of the full string theory at the boundary of the AdS space, can be quantized by imposing

$$\left[\mathcal{Z}^\Omega_\chi(\sigma,\tau), \bar{\mathcal{Z}}^{\Omega'}_{\chi'}(\sigma',\tau')\right]_{\tau=\tau'} = \delta^{\Omega\Omega'}\delta_{\chi\chi'}\delta(\sigma-\sigma')\,. \tag{3.14}$$

The modes of the twistors $\mathcal{Z}$ are nothing but the infinite set of oscillators used in [8, 5, 6] to construct representations of $\mathrm{SU}(2,2)$. Therefore, we have found states in string theory on AdS_5 which form representations of $\mathrm{SU}(2,2)$.

4. Supersymmetric model

Supertwistors [3] realize the full $\mathrm{SU}(2,2|4)$ algebra linearly. They contain two anti-commuting 2-component spinors (ξ, ε) in addition to the commuting spinors (λ, μ). The supergroup action on the fundamental representation of $\mathrm{SU}(2,2|4)$ is of the form:



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Here again one considers supertwistors[4] as fundamental, and the (x,θ)-superspace as a derived concept. The supertwistor $\mathcal{Z}$ and the metric now naturally extend to

$$\mathcal{Z} = \begin{pmatrix} \lambda_\alpha \\ \mu^{\dot\alpha} \\ \xi^i \\ \varepsilon^r \end{pmatrix}, \qquad H = \left(\begin{array}{cc|cc} & 1 & & \\ 1 & & & \\ \hline & & -1 & \\ & & & -1 \end{array}\right), \qquad \bar{\mathcal{Z}} = \mathcal{Z}^\dagger H\,. \tag{4.1}$$

We replace (3.6) by

$$\mu^{\dot\alpha} = -i\left(x^{\dot\alpha\alpha} + \frac{i}{2}(\bar\theta_i^{\dot\alpha}\theta^{\alpha i} + \bar\theta_r^{\dot\alpha}\theta^{\alpha r})\right)\lambda_\alpha \equiv -iz^{\dot\alpha\alpha}\lambda_\alpha \tag{4.2}$$

and similarly define θ by

$$\begin{aligned} \xi^i &= \theta^{\alpha i}\lambda_\alpha\,, \qquad i = 1,2\,; \\ \varepsilon^r &= \theta^{\alpha r}\lambda_\alpha\,, \qquad r = 1,2\,, \end{aligned} \tag{4.3}$$

so that the bilinear form $\bar{\mathcal{Z}}(1)\mathcal{Z}(2)$ is manifestly invariant under $\mathrm{SU}(2,2|4)$. An additional constraint comes from the reality of x^μ, which implies that the $\mathrm{SU}(2,2|4)$-invariant norm of $\mathcal{Z}$ vanishes.

The two-dimensional supertwistor model we would like to consider is

$$S = i\int d^2\sigma\left(\bar{\mathcal{Z}}_L\partial_-\mathcal{Z}_L + \bar{\mathcal{Z}}_R\partial_+\mathcal{Z}_R\right). \tag{4.4}$$

This world-sheet action is manifestly $\mathrm{SU}(2,2|4)$-invariant. The canonical quantization condition is

$$\left[\mathcal{Z}_\chi^\Omega(\sigma,\tau), \bar{\mathcal{Z}}_{\chi'}^{\Omega'}(\sigma',\tau')\right\}_{\tau=\tau'} = \delta^{\Omega\Omega'}\delta_{\chi\chi'}\delta(\sigma-\sigma')\,. \tag{4.5}$$

On-shell for left- and right-movers we may use an expansion

$$\mathcal{Z}_L = \sum_{n=-\infty}^{n=\infty} e^{-in(\tau+\sigma)}Z_{Ln}\,, \qquad \mathcal{Z}_R = \sum_{n=-\infty}^{n=\infty} e^{-in(\tau-\sigma)}Z_{Rn}\,. \tag{4.6}$$

The quantization condition for the left and right movers takes the form

$$\left[Z_{Lm}^\Omega, \bar{Z}_{Ln}^{\Omega'}\right\} = \delta^{\Omega\Omega'}\delta_{mn}\,, \qquad \left[Z_{Rm}^\Omega, \bar{Z}_{Rn}^{\Omega'}\right\} = \delta^{\Omega\Omega'}\delta_{mn}\,, \tag{4.7}$$

and the Hamiltonian reads

$$\mathcal{H} = i\int d\sigma\left(\bar{\mathcal{Z}}_L\mathcal{Z}_L' - \bar{\mathcal{Z}}_R\mathcal{Z}_R'\right) = \sum_{n=-\infty}^{\infty} n(\bar{Z}_{Ln}Z_{Ln} + \bar{Z}_{Rn}Z_{Rn})\,. \tag{4.8}$$

[4]Detailed notation for supertwistors will be given in [11].

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Thus we find that the world-sheet supertwistor theory (4.4) has all superoscillators which are required to construct the positive-energy unitary supermultiplets of the $\mathrm{SU}(2,2|4)$ superalgebra in a Fock space.[5] Also, the supertwistor action and the superoscillator construction lead to the short massless and massive supermultiplets [6] as well as to long and intermediate supermultiplets related to massive string states.

5. Physical states of the supertwistor model

To understand the connection between the two-dimensional supertwistor action and the oscillator construction of [6] it is useful to split the supertwistor into components. The lagrangean (4.4) is

$$\mathcal{L} = i\Big(\bar\lambda_L\partial_-\mu_L + \bar\mu_L\partial_-\lambda_L - \bar\xi_L\partial_-\xi_L - \bar\varepsilon_L\partial_-\varepsilon_L + \\ +\bar\lambda_R\partial_+\mu_R + \bar\mu_R\partial_+\lambda_R - \xi_R\partial_+\xi_R - \bar\varepsilon_R\partial_+\varepsilon_R\Big)\,. \tag{5.1}$$

The quantization condition in components for all left-movers $\bar Z_{Lm}, Z_{Ln}$ is

$$\begin{aligned} &[\lambda_{\beta m},\bar\mu^\alpha_n] = \delta^\alpha_\beta\delta_{mn}\,, \qquad &&[\mu^{\dot\beta}_m,\bar\lambda_{\dot\alpha n}] = \delta^{\dot\beta}_{\dot\alpha}\delta_{mn}\,,\\ &\{\xi^i_m,\bar\xi_{jn}\} = \delta^i_j\delta_{mn}\,, \qquad &&\{\varepsilon^r_m,\bar\varepsilon_{sn}\} = \delta^r_s\delta_{mn} \end{aligned} \tag{5.2}$$

and analogous expressions can be given for all right-movers $\bar Z_{Rm}, Z_{Rn}$. To connect this construction to the superoscillators of $\mathrm{SU}(2,2|2+2)$ we identify the set of pairs of superoscillators $(\xi_A,\xi^A,\eta_M,\eta^M)$ for each generation with our supertwistors as suggested by the model (4.4):

$$\begin{aligned} \xi_A &= \begin{pmatrix} a_i\\ a_\gamma\end{pmatrix} \Longleftrightarrow \begin{pmatrix}\lambda_\alpha\\ \xi^i\end{pmatrix}, \qquad & \xi^A &= \begin{pmatrix} a^i\\ a^\gamma\end{pmatrix} \Longleftrightarrow \begin{pmatrix}\bar\mu^\alpha\\ \bar\xi_i\end{pmatrix},\\ \eta_M &= \begin{pmatrix} b_r\\ \beta_x\end{pmatrix} \Longleftrightarrow \begin{pmatrix}\mu^{\dot\beta}\\ \varepsilon^s\end{pmatrix}, \qquad & \eta^M &= \begin{pmatrix} b^r\\ \beta^x\end{pmatrix} \Longleftrightarrow \begin{pmatrix}\bar\lambda_{\dot\alpha}\\ \bar\varepsilon_r\end{pmatrix}. \end{aligned} \tag{5.3}$$

Our model therefore has the complete spectrum of states obtainable by the oscillator method [5, 6]. All supermultiplets described in [5, 6] are characterized by the value of Z, which is an eigenvalue of the $\mathrm{U}(1)_Z$ generator commuting with all other generators of $\mathrm{SU}(2,2|4)$. The states with $Z=0$ form representations of the simple superalgebra $\mathrm{PSU}(2,2|4)$.

[5] We should note that noncompact groups admit also non-lowest weight type representations that can not in general be realized over a Fock space.

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For $Z = 0$ the following supermultiplets of states follow from the quantized action (4.4):

1. $P = 1$ *doubleton.* By choosing one pair of superoscillators, or one supertwistor, one can construct the ultra-short doubleton supermultiplets. The CPT-self-conjugate doubleton of $\mathrm{SU}(2,2|4)$ corresponds to the $\mathcal{N} = 4$ supersymmetric Yang-Mills supermultiplet in $d = 4$. It is the unique irreducible doubleton supermultiplet with $Z = 0$. It is known to decouple from the Kaluza-Klein spectrum [5, 13]. It corresponds to a gauge supermultiplet which couples only at the boundary of the AdS space.

2. $P = 2$ *'massless' AdS supermultiplets.* These include the $\mathcal{N} = 8$ graviton supermultiplet on AdS_5, which appears in the tensor product of the two CPT-self-conjugate doubletons. The full description of generic supermultiplets built out of two generations of superoscillators is given in [5, 6]. The general massless supermultiplets with $Z = 0$ are those given in Table 12 of the first reference of [6] for which $j_L = j_R$. They have spin range between 2 and 4. The ten-dimensional interpretation of the intermediate and long massless supermultiplets is an interesting open problem. We see that we need two supertwistors for this purpose. Obviously the (σ-independent) zero modes of the left and right-moving twistors give a natural choice of oscillators for $P = 2$ supermultiplets.

3. All $P > 2$ short massive supermultiplets of spin range two, which form an infinite tower of states with a 'massless' graviton supermultiplet of $\mathcal{N} = 8$ AdS_5 ($P = 2$) sitting at the bottom. These states are the irreducible "CPT–self-conjugate" short supermultiplets of IIB supergravity compactified on S^5 [5] ('massive' Kaluza-Klein modes). The lowest weight vector of these supermultiplets is the vacuum state annihilated by all the super-oscillators (P generations).

4. All $P > 2$ intermediate massive supermultiplets of spin range less than 8 and long massive supermultiplets of spin range 8 with $Z = 0$. The supertableaux of lowest weight vectors of supermultiplets with $Z = 0$ have equal number of boxes with respect to both $\mathrm{SU}(2|2)_L$ and $\mathrm{SU}(2|2)_R$. The long massive supermultiplets appear for $P \geq 4$.

For $Z \neq 0$ the following supermultiplets of states follow from the quantized action (4.4):

1. The CPT non-self-conjugate doubleton supermultiplets of [6] with $Z = \pm J$. They are constructed from one generation of the supertwistors ($P = 1$). These are massless representations of the $\mathcal{N} = 4$ Poincaré superalgebra in $d = 4$.

2. The intermediate and long massless supermultiplet for $P = 2$ and $Z \neq 0$ classified in [6].

3. The novel short supermultiplets with $P \geq 2$ and $Z \neq 0$, which are not CPT–self-conjugate. They are conjectured in [6] to include 1/4 BPS states of Yang-Mills theory discovered in [14] and may also belong to the spectrum of (p, q) IIB strings. These multiplets also naturally come from quantized supertwistors of our model and here again we have an infinite number of such states with $P \geq 2$ due to the infinite number of modes of the 2-dimensional theory.

4. Intermediate and long massless supermultiplets with $P = 2$ that are not CPT–self-conjugate, described in [6].

5. Intermediate and long massive supermultiplets with $Z \neq 0$.

6. The descendants of the massive string states

A subset of the supermultiplets discussed above come directly from the ten-dimensional supergravity compactified on $AdS_5 \times S^5$. These are all massless in the ten-dimensional sense and therefore belong to supermultiplets with spin range 2 and $Z = 0$. The CPT self-conjugate doubleton supermultiplet, which has spin range 1, decouples from the spectrum as gauge modes in the bulk.

One would also like to identify the towers of massive supermultiplets on $AdS_5 \times S^5$ that come from the massive string states. In $d = 10$, these states are not described by classical supergravity and it is this tower of massive states which makes all the difference between field theory and string theory in $d = 10$.

Obviously, since the supertwistors are the fundamentals of the $\mathrm{SU}(2, 2|4)$ one should be able to find such massive states from our quantized action, or equivalently in the framework of the superoscillator construction [5, 6]. The massive string states are generically expected to belong to long multiplets with spin range 8, though some of them may belong to intermediate massive multiplets. In fact, such massive supermultiplets can easily be identified. Now the regular massive states of the IIB superstring over $AdS_5 \times S^5$ are expected to fall into massive supermultiplets with $Z = 0$ and those of (p, q) strings will in general have $Z \neq 0$. A typical example of a long massive supermultiplet is provided by choosing a lowest weight vector of $\mathrm{SU}(2, 2|4)$ whose supertableau with respect to $\mathrm{SU}(2|2)_L \times \mathrm{SU}(2|2)_R$ is of the form:

$$\left| \; \underbrace{\begin{array}{|c|c|c|c|}\hline \diagup & \diagup & \cdots & \diagup \\ \hline \diagup & \diagup & \cdots & \diagup \\ \hline \end{array}}_{n}\!\underbrace{\begin{array}{|c|c|c|}\hline \diagup & \cdots & \diagup \\ \hline \end{array}}_{2j_L} \;,\; \underbrace{\begin{array}{|c|c|c|c|}\hline \diagup & \cdots & \diagup \\ \hline \diagup & \cdots & \diagup \\ \hline \end{array}}_{m}\!\underbrace{\begin{array}{|c|c|c|}\hline \diagup & \cdots & \diagup \\ \hline \end{array}}_{2j_R} \; \right\rangle , \tag{6.1}$$

where we take

$$P \geq 4 \,; \qquad j_L, j_R \geq 2 \,, \qquad n, m \geq 2 \,. \tag{6.2}$$

The lowest weight vector above leads to a supermultiplet of fields whose $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ quantum numbers range between

$$(j_L - 2, j_R - 2)\,, \qquad \longrightarrow \qquad (j_L + 2, j_R + 2)\,, \tag{6.3}$$

which corresponds to a spin range of 8 and has $Z = n - m + j_L - j_R$. By choosing $j_L = j_R + (m - n)$ we get long massive supermultiplets with $Z = 0$ of corresponding to the massive string states of the regular IIB superstring. Interestingly, both the lowest spin state and the highest spin state have the same AdS energy $E = m + n + j_L + j_R + P$. A detailed study and classification of intermediate and long massive supermultiplets of $\mathrm{SU}(2,2|4)$ will be given elsewhere [15].

The supermultiplets of spin range 8 can be constructed starting with 4 generations of supertwistors. This is precisely what one would have expected: neither a superparticle action with $P = 1$ nor the zero modes of the 2-dimensional action (both left and right movers with $P = 2$) would give us massive string states. To have $P \geq 4$ we really need the σ-dependence in the supertwistor, as in the usual string action in the flat superspace.

7. Discussion

In conclusion, our world-sheet supertwistor action may be viewed as a quadratic quark-type action for $\mathrm{SU}(2,2|4)$ supergroup. The analogy here is that from the fundamental representations (quarks, anti-quarks) of SU(3) one can obtain all the other representations (mesons, baryons, etc.) by a simple tensoring procedure. Here we have two infinite towers of generations of fundamentals of $\mathrm{SU}(2,2|4)$, which provide a complete set of states of the theory which can be described in a Fock space.

One should point out that the supertwistor action (4.4) is not derived from the GS string theory on AdS_5. The relation between these two theories is more subtle, as it is the case of linear and non-linear sigma models in general.

The simple supertwistor action, as we have seen, is capable of explaining the spectrum of all positive energy UIR's of $\mathrm{SU}(2,2|4)$. Some of these representations can be identified with Yang-Mills theory, some are states of supergravity including KK modes, and some novel representations do not seem to have a clear identification in any dynamical theory studied so far. If indeed they correspond to the (p,q) states of IIB superstring theory, as conjectured in [6], they may not be captured by the GS superstring action on $AdS_5 \times S^5$. Rather one may try to study the $\mathrm{SL}(2,\mathbf{Z})$ invariant string theory [16]. The action of Townsend and Cederwall is also known in an arbitrary background of IIB supergravity. Therefore we may apply the supercoset methods and gauge-fix κ-symmetry in this string action. The states of this theory will be $\mathrm{SU}(2,2|4)$-symmetric since this will be a symmetry of the background. But the $\mathrm{SL}(2,\mathbb{Z})$ symmetry of this string theory suggests the presence of (p,q) states.

It is possible therefore that we may find some interesting relation of this $\mathrm{SL}(2,\mathbb{Z})$ symmetric string theory to the supertwistor world-sheet model presented in this paper.

Finally, we have shown here that the long supermultiplets of the $\mathrm{SU}(2,2|4)$ associated with the massive string supermultiplets are also present in the spectrum of the supertwistor model. More detailed studies of such supermultiplets and their relation to ten-dimensional massive string states will be important.

Thus, the fact that the supertwistor world-sheet action (4.4) codifies all information on positive energy UIR's of $\mathrm{SU}(2,2|4)$ which can be obtained by the superoscillator method of [6] suggests that this may be an interesting direction to develop. One is even tempted to conclude that (super) twistors on the two-dimensional world-sheet may be fundamental whereas the (super) space-time is a derivable concept.

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